Quantification of Reinvestment Risk in the Private Investment Portfolio

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It has long been an article of faith in the private investment industry that an investor would rather have 18% returns over 10 years than 35% returns over two years. This article of faith is based on the intuitive notion that reinvestment risk will make it difficult to achieve the earlier, higher IRR over the longer time period.

The issue of reinvestment risk is becoming an important portfolio management issue for many reasons, among the most important of which is the increased allocation to private investments in many of the largest public and private pension funds, as well as in many endowments and foundations. Because institutional investors do not know exactly when funds will be called after entering into a private investment fund commitment (or in what amounts) and because they also do not know exactly when they will receive distributions from existing private investment funds (or in what amounts), a private investment program is a bewilderingly difficult cash management problem. The larger the allocation to private investments, the larger the cash management problem. Quantifying reinvestment risk in the private investment portfolio is one important step toward addressing cash management risk and return in the overall portfolio.

The purpose of this article, then, is to quantify reinvestment risk—the risk that a distribution, when received and reinvested, will not achieve the returns expected upon the making of the original investment.

RISK ASSUMING REINVESTMENT INTO A LIQUID ASSET CLASS

In order to quantify the reinvestment risk inherent in any distribution that is to be reinvested into a liquid asset class, we need to know the following:

1. The returns available in the liquid reinvestment vehicle (r_1), which for purposes of this article I have assumed to be the S&P 500 long-term return.[1]
2. The standard deviation of the liquid reinvestment vehicle (σ_1), which for this purpose I have assumed to be the long-term standard deviation of the S&P 500.[2]
3. The year in which the private investment is realized and distributed (m).
4. The time horizon expected at the original private investment date (i.e., draw-down date) (n).
5. The IRR expected for the private investment asset class in question over the expected time horizon (r_2).

Given these inputs, I propose to ask (and, I hope, answer) the following question:

- If the investor invests $1 at time 0, how many dollars must be distributed at time m such that the lower boundary of the returns on the amount distributed, when the amount distributed is rein-

EXHIBIT 1
IRR Indifference Curve Assuming Reinvestment in Liquid Asset

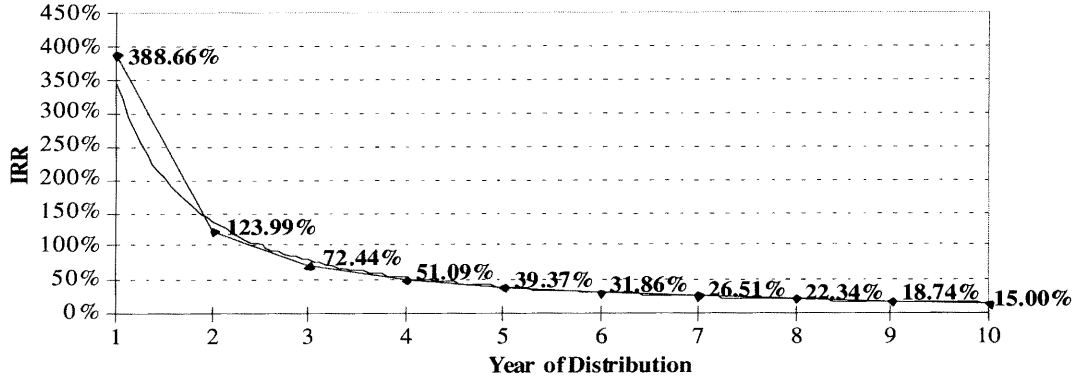
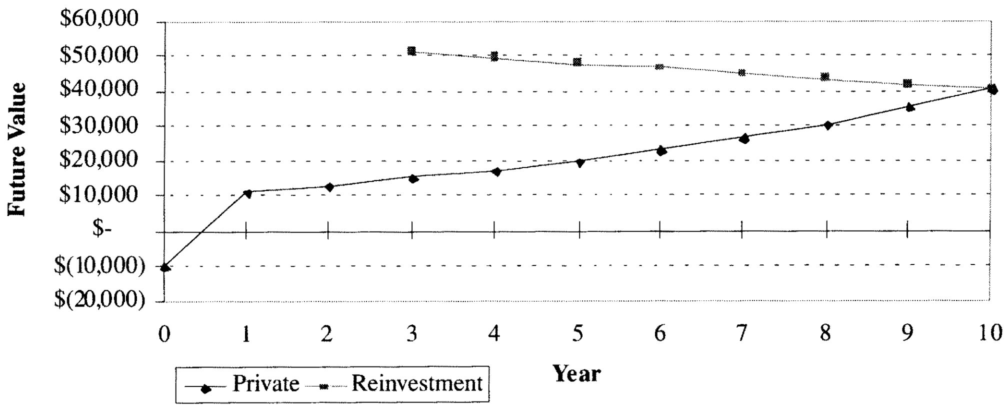


EXHIBIT 2
Value of Original Investment After Ten Years



vested at rate r_1 (here, the long-term S&P 500 rate), will at least equal the expected value of the investment had it remained invested until time n?

As calculated in Appendix A, the answer is:

$$x = \frac{(1+r_2)^n \left(1+\sigma_{\bar{r}_1}\right)^{n-m}}{(1+r_1)^{n-m}} \tag{1}$$

Or, put somewhat differently,

- What is the rate of return (r_3) required to make the lower boundary of the future value of the amount distributed equal to the expected value of the original amount invested over the time horizon of the investment?

As calculated in Appendix A, the answer is:

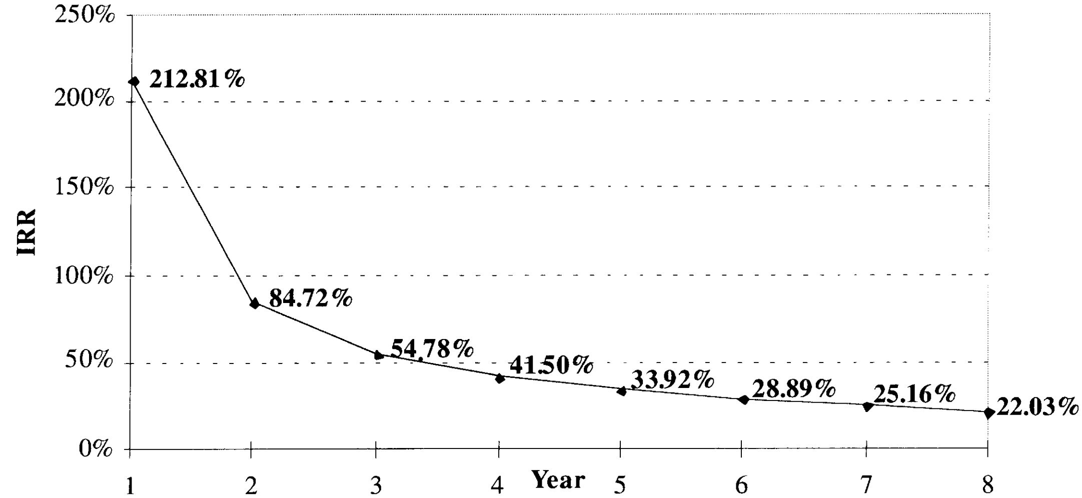
$$r_3 = \sqrt[m]{\frac{(1+r_2)^n \left(1+\sigma_{\bar{r}_1}\right)^{n-m}}{(1+r_1)^{n-m}}} - 1 \tag{2}$$

Exhibit 1 illustrates the use of Equation (1) over an intended holding period of 10 years, assuming the following:

$r_1 = 0.10$
$\sigma_1 = 0.39$
$r_2 = 0.15$

Exhibit 1 can be read to mean that there is an 84% probability (i.e., the 68% probability associated with one standard deviation above and below the mean plus the 16% upside tail) that the IRR indicated at each data point is the return necessary at that date to insure that the proceeds

EXHIBIT 3
Reinvestment in Illiquid Asset, Delay Period: One Year



distributed on that date will, at the end of 10 years from the original investment date, be at least equal to the amount that would have been realized had the amount distributed remained invested until the end of the tenth year at a 15% IRR.

Exhibit 2 uses Equation (2) to demonstrate that a distribution received in the third year from a private investment partnership would need to have appreciated from, perhaps, \$10,000 to \$51,247 in order for the lower border of the future value of the reinvestment of the distribution in the S&P 500 to be at least equal to the expected future value of the original investment after 10 years.

Equations (1) and (2) are sufficiently general to be used with any combinations of liquid reinvestment asset classes, assuming only that the liquid reinvestment asset class into which distributions are reinvested is sufficiently liquid to enable the investor to invest the entire distribution the date it is received.

RISK ASSUMING REINVESTMENT INTO AN ILLIQUID ASSET CLASS

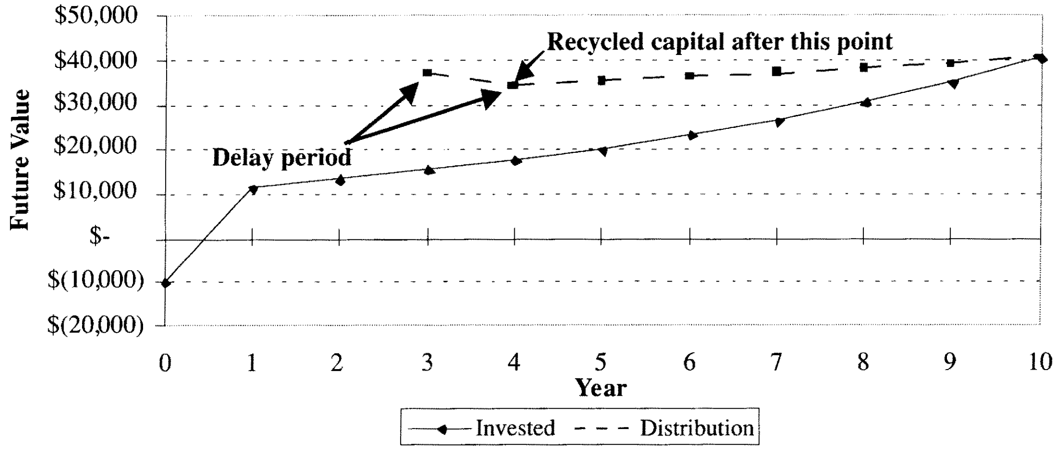
If private investment distributions are recycled back into illiquid private investments, a more detailed analysis is necessary. In addition to the factors detailed above, the analysis of the recycling of private investment distributions into further private investments requires the analyst to take into account the time required for the recycling to occur, the investment return likely to be earned during the delay period, and the risk of holding the investment during the delay period.

The delay period involved is a direct result of the nature of the private investment industry. Most private investors commit to limited partnerships that draw capital over some acquisition period, usually three to five years. If an investor receives a distribution from one private investment partnership, there may be a lengthy delay before the funds received are called and invested by a second private investment partnership. During the investor's holding period, these funds must be invested to earn some appropriate return. However, the higher the return during the delay period the higher the risk that at the future date when the funds are to be reinvested in an illiquid asset class, the value of the investment may be materially higher or lower than the amount called.

In order to quantify the reinvestment risk inherent in any distribution that is to be reinvested into an illiquid private investment (i.e., recycled), we need to know the following:

1. The year in which the private investment is realized and distributed (m_1).
2. The returns available in the most likely liquid reinvestment vehicle into which the distribution will be invested during the delay period (r_1), which for purposes of this article I have assumed to be 8%.
3. The standard deviation of the most likely liquid reinvestment vehicle into which the distribution will be invested during the delay period (σ_1), which for this purpose I have assumed to be 17%.

EXHIBIT 4
Indifference Curve Assuming One-Year Delay



4. The year in which the distribution is drawn down by the second private investment (m_2).
5. The time horizon expected at the original private investment date (i.e., the draw-down date in the first fund) (n).
6. The IRR expected for the second private investment class in question over the remaining time horizon (r_2).
7. The IRR expected for the original private investment asset class in question over the expected time horizon (r_3).
8. The standard deviation of the second private investment class in question over the remaining time period (σ_2), here assumed to be the long-term standard deviation calculated for realized private investments.[3]

Given these inputs, I propose to ask the following questions:

- If an investor invests \$1 at time 0, how many dollars must be distributed at time m_1 such that the lower boundary of the returns on the amount distributed, when the amount distributed is invested in a liquid asset until time m_2 (where $m_2 - m_1$ is the delay period) at r_1 with standard deviation σ_1 will, when called by and reinvested in a second private partnership with return r_2 and standard deviation σ_2, at least equal the expected value of the investment had it remained invested in the original partnership at rate of return r_3 until time n?

As calculated in Appendix B, the answer is:

$$x = \frac{(1+r_3)^n \left(1+\sigma_{\bar{r}_1}\right)^{m_2-m_1}}{(1+r_1)^{m_2-m_1}} \frac{\left(1+\sigma_{\bar{r}_2}\right)^{n-m_2}}{(1+r_2)^{n-m_2}} \tag{3}$$

Or, put somewhat differently,

- What is the rate of return (r_4) required to make the lower boundary of the future value of the amount distributed equal to the expected future value of the original amount invested over the time horizon of the investment?

As calculated in Appendix B, the answer is:

$$r_4 = \sqrt[m_1]{\frac{(1+r_3)^n \left(1+\sigma_{\bar{r}_1}\right)^{m_2-m_1}}{(1+r_1)^{m_2-m_1}} \frac{\left(1+\sigma_{\bar{r}_2}\right)^{n-m_2}}{(1+r_2)^{n-m_2}}} - 1 \tag{4}$$

Exhibit 3 illustrates the use of Equation (4) over an intended holding period of 10 years, assuming the following:

$r_1 = 0.08$
$\sigma_1 = 0.17$
$r_2 = 0.15$
$\sigma_2 = 0.29$
$r_3 = 0.22$
$m_2 - m_1 = 1$ year

EXHIBIT 5
Differential Value of Delay Risk

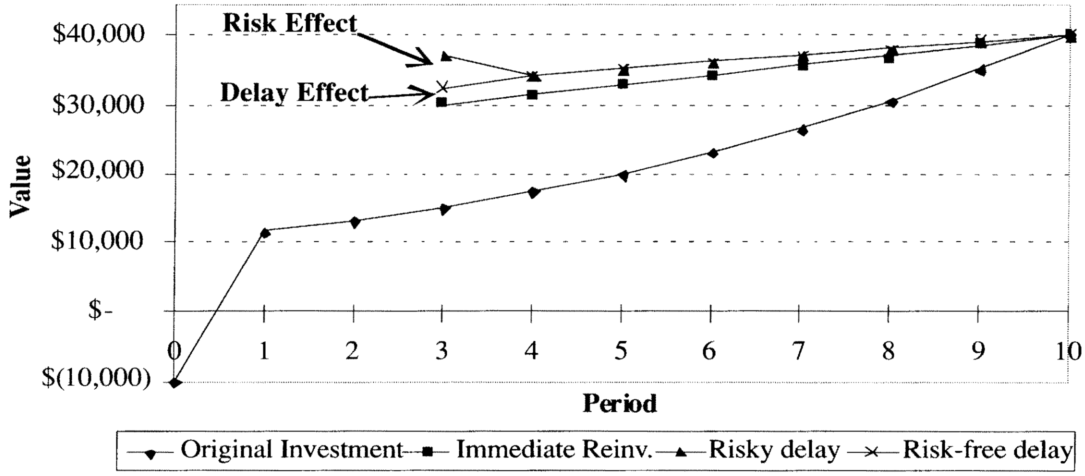


Exhibit 3 can be read to say that the return indicated at each point is the return required in order to give the investor an 84% certainty (i.e., the 68% probability associated with one standard deviation above or below the mean plus the 16% upside tail) that the amount distributed on that date, after being parked for the one-year delay period in a liquid investment yielding 8% with a standard deviation of 17%, be reinvested (i.e., recycled by being drawn down) in a second private investment with the same 15% return and 29% standard deviation as the first private investment.

Using Equation (3) we can calculate that a distribution received in the third year from a private investment partnership would need to have appreciated from, perhaps, $10,000 to $37,079 in order for the lower border of the future value of the parking of the distribution for one year in a short-term bond investment yielding 8% with a standard deviation of 17%, followed by investing that sum such that the lower border of the future value of the investment in the second private investment fund, with investment characteristics the same as the first fund (a 15% expected return with a standard deviation of 29%), will be at least equal to the expected future value of the original investment after 10 years.

USE OF REINVESTMENT RISK ANALYSIS AS AN INVESTMENT SCREEN

In addition to its utility as a cash management tool, it is possible to use reinvestment risk as an investment screen to help the analyst determine the merits of a private investment opportunity. For example, in analyzing a prospective private investment fund with a particular historical return, what weighted average holding period would have been necessary to neutralize the reinvestment risk inherent in its historical distribution pattern? Put another way, over time has a particular private investment group demonstrated a holding period long enough to at least neutralize the reinvestment risk resulting from its weighted average holding period, given the risk, return, and delay period inherent in reinvesting the candidate group's distributions in the same asset class?

One simple way to accomplish this analysis is to calculate the indifference curve shown in Exhibit 1 (assuming reinvestment in a liquid asset class) or Exhibit 3 (assuming reinvestment in an illiquid asset class), using assumptions appropriate for the opportunity, then to note where the candidate firm's long-term return appears on the curve. If the group's weighted average holding period is equal to or greater than the return associated with the return shown on the indifference curve, then the investors have historically been compensated properly for the reinvestment risk involved.

For example, given the assumptions used for Exhibit 4 (i.e., a delay period of one year, during which time the distribution is invested at 8% with a standard deviation of 17%, followed by reinvestment at 15% with a standard deviation of 29%), and further assuming that a private investment fund opportunity in the past has returned about 25%, then the weighted average holding period should have been seven years or longer in order to compensate for the reinvestment risk involved.

Looked at from the other direction, if the private investment opportunity has had an historical weighted average holding period of seven years, then it should have had an IRR of at least 25% in order to compensate the investor for the reinvestment risk associated with the timing of the distributions.

Of course, all these calculations should also be done on a prospective basis using the analyst's assumptions about the private investment asset class involved and about the particular investment as well. The analyst can then weigh carefully the likelihood that the investment under consideration actually will be able to generate the necessary returns or higher and/or have the required holding period or longer.

CASH MANAGEMENT RISK ATTRIBUTION

The analysis outlined above can be modified and extended to determine the cost to the portfolio manager of the delay period (i.e., the length of time between the receipt of a distribution and the time the funds are reinvested in the same illiquid asset class), and the cost of the delay risk (the return needed to compensate the portfolio manager for the risk that the funds temporarily invested pending reinvestment in the same asset class may decline in value during the delay period).

Thus, in Exhibit 5 the difference between the size of the investment required to temporarily invest distributed funds in a risk-free asset, and the amount required to invest the funds in a risky but higher-yielding temporary investment in order to achieve the same terminal value, is termed the *risk effect*; while the difference between reinvesting immediately in the same asset class and temporarily investing first in a risk-free asset is termed the *delay effect*. The return differentials necessary to result in the amounts of the distributions shown in the above graph are the returns necessary to compensate the portfolio manager for these two cash management problems: the risk that amounts distributed cannot immediately be reinvested in the same asset class to generate the same returns (the delay effect), and the risk involved in investing in risky assets during the delay period until the funds can be reinvested in the same asset class (the risk effect). These two effects, when added, comprise the total cash management risk involved in investing in the private markets.

The return related to delay risk can be calculated by finding the difference between the return required to compensate the investor if a distribution can be reinvested immediately upon receipt in the asset class that generated it (using Equation (2) and assuming that the distribution occurs at time m_1, r_3 is the return originally sought, r_2 is the return available upon immediate reinvestment in the same asset class and given a reinvestment period of n-m_1) and the amount required to compensate the investor if a delay period $m_2 - m_1$ is required before reinvesting in the same asset class [using Equation (4) above]:

$$r_{delay} = r_L - r_F \text{ where} \quad (5)$$

- r_L is the return needed to compensate the investor for reinvestment risk given immediate reinvestment in the same asset class at the same return [Equation (2)], and
- r_F is the return needed to compensate the investor for reinvestment risk given delayed reinvestment with the original distribution parked in the interim in a zero-coupon Treasury maturing the date of reinvestment [Equation (4)].

The return related to the risk effect can be calculated by finding the difference between the return necessary to compensate the investor given a delayed reinvestment with the funds parked in a risk-free investment vs. the return necessary to compensate the investor given a delayed reinvestment with the funds parked in a risky investment:

$$r_{risk} = r_{RiskyDelay} - r_F \text{ where} \quad (6)$$

- $r_{RiskyDelay}$ is the return needed to compensate the investor for reinvestment risk given delayed reinvestment with the original distribution parked in the interim in a risky fixed income investment, and
- r_F is the variable defined in Equation (5).

Using the same assumptions incorporated throughout this article for the delay period (one year), the risky interim investment (8% return with a standard deviation of 17%) and the private investment (15% return with a 29% standard deviation) and assuming a 5% risk-free one-year Treasury zero-coupon investment as the alternative, the return necessary to compensate for the delay risk in the graph above is 3.9% (48.3% to compensate the investor for a risk-free one year delay prior to reinvestment minus 44.4% to compensate for the reinvestment risk with immediate reinvestment in the private markets); and the return necessary to compensate for the risky interim investment is 6.5% (54.8% to compensate for a risky investment during the one year delay minus 48.3% to compensate for a risk-free delay investment).

The total risk spread of 10.4% (3.9% + 6.5%) is the excess return needed to compensate for the cash management problems that give rise to the delay in the first place. The remaining return compensates the investor for the risk that the reinvested funds will perform at the lowest expectations for the asset class into which the funds are reinvested over the time period originally contemplated.

CONCLUSION: INVESTMENT POLICIES IMPLIED BY THE ANALYSIS

There are at least five investment policies implied by the above analysis. First, the delay period between receipt of a private investment distribution and its reinvestment in a succeeding private investment should be as short as possible. The longer the delay period, assuming a private investment distribution is parked in a risky asset, the more the cash management risk from the overall portfolio perspective.

Second, the parking vehicle used for the delay period should be as low risk as possible. Use of a parking vehicle that is riskier than the original investment almost guarantees a greater reinvestment risk, since the principal to be reinvested is at risk during the delay period. In other words, effective cash management is extremely important in minimizing the reinvestment risk in a private investment portfolio.

It is interesting to note that the investment record of Warren Buffett, for example, has in some measure been attributed by a number of observers to the length of his holding period, which has simultaneously had the effect of avoiding reinvestment risk and the taxation of capital gains. These two effects, when taken together, have resulted in maximization of the effect of his security selection. The same conclusions apply to private investment groups associated with the leveraged build-up, which usually requires a lengthy holding period.

Third, the reinvestment risk analysis outlined above suggests that longer-term private investments may well provide an overall return superior to quick-turn private investments when reinvestment risk is taken into account. Most funds cannot generate returns sufficient to compensate institutional investors for the implied reinvestment risk involved where the holding period is extremely short.

Fourth, it may be advantageous to allow the general partner to recycle capital in order to avoid the reinvestment risk associated with short-term gains. It is important to note in this context, however, that simply permitting the fund general partner to recycle capital does not obviate the reinvestment problem. It simply moves the reinvestment responsibility to the fund general partner. If the fund general partner is more likely to employ the gains harvested from an earlier investment prior to the time a distribution of the same gains would be drawn by some other fund general partner in the institutional investor's portfolio, then a recycling program makes sense because it minimizes the delay period. Otherwise, the institutional investor's duties and responsibilities include rigorous cash management in addition to private investment portfolio risk-adjusted return maximization. As noted above, the cash management portion of the institutional direct investor's responsibilities must be designed to reduce reinvestment risk as far as possible by minimizing the risk of loss of value during the delay period prior to reinvestment and by minimizing the delay period itself.

Fifth, and finally, reinvestment risk analysis can and should be used as an investment screen and to determine risk attribution in reviewing the track record of private investment managers. The analyst's purpose in examining the reinvestment risk posed by a particular manager's historical record is to determine whether the returns generated were sufficient to compensate the investors for the reinvestment risk inherent in short holding periods.

APPENDIX A

As adapted from equations well established in the literature of finance,[4] the upper and lower boundaries of the future value of an amount invested at time m until time n (in this case, the amount distributed to the limited partners at time m, invested until the end of the originally planned holding period n) at a 68% probability are as follows:

$$FV_{\max} = \left(1 + r_1\right)^{n-m}\left(1 + \sigma_{\bar{r}_1}\right)^{n-m} \tag{A-1}$$

where

$$\sigma_{\bar{r}_1} = \frac{\sigma_{r_1}}{\sqrt{n-m}}$$

$$FV_{\min} = \frac{(1+r_1)^{n-m}}{\left(1+\sigma_{\bar{r}_1}\right)^{n-m}} \quad \text{(A-2)}$$

It is obvious that the amount originally invested would grow to a future value over the term of the investment as follows:

$$FV = (1+r_2)^n \quad \text{(A-3)}$$

Thus, the answer to the first question set out above is that the minimum future value (i.e., the lower boundary of the future value) of the amount received as a distribution (after being invested in the most likely reinvestment vehicle) must equal the expected future value of the amount originally invested over the time period of the investment at the return rate originally expected:

$$x\left(\frac{(1+r_1)^{n-m}}{\left(1+\sigma_{\bar{r}_1}\right)^{n-m}}\right) = (1+r_2)^n \quad \text{(A-4)}$$

and thus

$$x = \frac{(1+r_2)^n\left(1+\sigma_{\bar{r}_1}\right)^{n-m}}{(1+r_1)^{n-m}} \quad \text{(A-5)}$$

The answer to the second question set out above is that the return required to generate the dollars returned in the preceding equation is the discount rate necessary to make the future value of the amount invested equal to the present value of the investment:

$$1 = \frac{x}{(1+r_3)^m} \quad \text{(A-6)}$$

$$(1+r_3)^m = x \quad \text{(A-7)}$$

$$r_3 = \sqrt[m]{x} - 1 \quad \text{(A-8)}$$

By combining the answers to these two questions and substituting the value of x determined above we can obtain a generalized solution to the problem:

$$r_3 = \sqrt[m]{\frac{(1+r_2)^n\left(1+\sigma_{\bar{r}_1}\right)^{n-m}}{(1+r_1)^{n-m}}} - 1 \quad \text{(A-9)}$$

APPENDIX B

Equations (A-1) and (A-2) in Appendix A must be modified in the context of recycling capital into a second private investment partnership by calculating, first, the upper and lower boundaries to be expected when investing the distribution from the first private partnership in some liquid reinvestment vehicle until the funds distributed can be called and reinvested by a second private partnership; and second, the upper and lower boundaries to be expected of the investment in the second private partnership. Assuming the future values calculated have an 84% probability (i.e., the 68% probability associated with one standard deviation above or below the mean plus the 16% upside tail), the appropriate formulas to determine the upper and lower boundaries of the future value of the distribution from the point at which it is received to the point at which it is recycled (i.e., at the end of the delay period) are as follows:

$$FV_{\max m_2} = (1+r_1)^{m_2-m_1}\left(1+\sigma_{\bar{r}_1}\right)^{m_2-m_1} \quad \text{(A-10)}$$

where

$$\sigma_{\bar{r}_1} = \frac{\sigma_1}{\sqrt{m_2-m_1}}$$

$$FV_{\min m_2} = \frac{(1+r_1)^{m_2-m_1}}{\left(1+\sigma_{\bar{r}_1}\right)^{m_2-m_1}} \quad \text{(A-11)}$$

As a second step, the same formulas can be used to calculate the upper and lower boundaries to be expected after the amount originally distributed at time m_1 is reinvested at its future value at time m_2 at r_2 with standard deviation σ_2:

$$FV_{\max} = FV_{\max_{m_2}}(1+r_2)^{n-m_2}\left(1+\sigma_{\bar{r}_2}\right)^{n-m_2} \quad \text{(A-12)}$$

$$FV_{\min} = FV_{\min_{m_2}}\frac{(1+r_2)^{n-m_2}}{\left(1+\sigma_{\bar{r}_2}\right)^{n-m_2}} \quad \text{(A-13)}$$

where

$$\sigma_{\bar{r}_2} = \frac{\sigma_2}{\sqrt{n - m_2}}$$

Substituting (A-10) into (A-12) and (A-11) into (A-13) results in the following equations, which describe the upper and lower boundaries over the combined remaining holding period at the distribution date (*viz.*, the delay period plus the remaining holding period after the point of reinvestment until the original time horizon n):

$$FV_{\max} = (1+r_1)^{m_2-m_1}\left(1+\sigma_{\bar{r}_1}\right)^{m_2-m_1}(1+r_2)^{n-m_2}\left(1+\sigma_{\bar{r}_2}\right)^{n-m_2} \quad \text{(A-14)}$$

$$FV_{\min} = \frac{(1+r_1)^{m_2-m_1}}{\left(1+\sigma_{\bar{r}_1}\right)^{m_2-m_1}} \frac{(1+r_2)^{n-m_2}}{\left(1+\sigma_{\bar{r}_2}\right)^{n-m_2}} \quad \text{(A-15)}$$

As before, the amount originally invested would grow to a future value over the term of the investment as follows:

$$FV = (1+r_3)^n \quad \text{(A-16)}$$

Thus, the minimum future value (i.e., the lower boundary of the future value) of the amount received as a distribution, after being invested in a liquid investment at r_1 with standard deviation σ_1 during the delay period and then reinvested in a private partnership with return r_2 and standard deviation σ_2, must equal the expected future value of the amount originally invested over the time period of the investment at the return rate originally expected:

$$x\left(\frac{(1+r_1)^{m_2-m_1}}{\left(1+\sigma_{\bar{r}_1}\right)^{m_2-m_1}} \frac{(1+r_2)^{n-m_2}}{\left(1+\sigma_{\bar{r}_2}\right)^{n-m_2}}\right) = (1+r_3)^n \quad \text{(A-17)}$$

$$x = \frac{(1+r_3)^n\left(1+\sigma_{\bar{r}_1}\right)^{m_2-m_1}}{(1+r_1)^{m_2-m_1}} \frac{\left(1+\sigma_{\bar{r}_2}\right)^{n-m_2}}{(1+r_2)^{n-m_2}} \quad \text{(A-18)}$$

The return required to generate the dollars returned in the preceding equation is the discount rate necessary to make the future value of the amount invested equal to the present value of the investment:

$$1 = \frac{x}{(1+r_4)^{m_1}} \quad \text{(A-19)}$$

$$(1+r_4)^{m_1} = x \quad \text{(A-20)}$$

$$r_4 = \sqrt[m_1]{x} - 1 \quad \text{(A-21)}$$

By substituting the value of x determined in Equation (A-20) into Equation (A-18), we can obtain a generalized solution to the problem:

$$r_4 = \sqrt[m_1]{\frac{(1+r_3)^n\left(1+\sigma_{\bar{r}_1}\right)^{m_2-m_1}}{(1+r_1)^{m_2-m_1}} \frac{\left(1+\sigma_{\bar{r}_2}\right)^{n-m_2}}{(1+r_2)^{n-m_2}}} - 1 \quad \text{(A-22)}$$

ENDNOTES

[1]Ibbotson & Sinquefield.

[2]Ibid.

[3]See A. Long [Summer 1999], for one approach to determining the interperiod standard deviation of a private investment portfolio based on the spread of the outcomes of its individual investments.

[4]See Bodie, Kane, and Marcus [1989] citing the Merton and Samuelson article, "Fallacy of the Log-Normal Approximation to Optimal Portfolio Decision-Making Over Many Periods" in *Risk and Return in Finance*, Volume I (Cambridge, Mass: Ballinger Publishing Co., 1977), edited by I. Friend and J. Bicksler.

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