Asset Allocation Effects of Adjusting Alternative Assets for Stale Pricing

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Investors in alternative asset classes such as private equity and hedge funds have long had difficulty applying traditional models for making asset allocation decisions. The optimization techniques of modern portfolio theory rely heavily on a trio of descriptive statistics: mean, variance, and covariance. For most traditional asset classes, the abundance of historical data provides a guide for estimating these parameters. However, for some alternative asset classes estimating these characteristics is not always straightforward.

Although time series of returns for both private equity and hedge funds are available from reputable sources, reported returns can be misleading. Alternative asset returns can exhibit low volatility and low correlations with publicly traded asset classes. This suggests that they are potentially diversifying assets and incremental allocations to alternative investments may decrease overall portfolio risk. The standard deviations and correlations of reported alternative asset indices, however, cannot be taken at face value. Partnerships holding illiquid securities are valued infrequently and are based on appraised values, so positions are not marked-to-market. This "stale pricing" dampens actual volatility.

The recent literature has confirmed that stale pricing exists in both hedge funds and private equity. Asness, Krail, and Liew [2001] explore the ability of hedge fund managers to smooth return streams, concluding that hedge funds appear to price their securities with a lag. Anson [2002] studies the same topic in private equity, and concludes that there is empirical evidence suggesting managed pricing does occur. He finds that general partners implement the "rule of conservatism," and mark down valuations more aggressively than they mark them up.

It is possible to empirically estimate the true volatility of alternative assets. We hypothesize that there exists an underlying process of returns that could be measured if positions were continually marked-to-market as in the public markets. We call this the "economic process" of returns. The reported index returns, based on stale prices, represent a "smoothed process" of returns. The volatility and correlation of the economic process represent actual economic events and are relevant to investors. Others have used a similar theoretical underpinning to adjust alternative asset returns for the effects of stale pricing. Asness, Krail, and Liew [2001] estimate the true correlation between hedge fund styles and the public markets in the presence of stale prices due to illiquidity or managed pricing using a multi-period regression approach. Gompers and Lerner [1997] employ an interperiod valuation approach to estimate the pricing activity of stale private equity positions, using public market activity as a proxy for unobservable private market pricing. To circumvent the issues of stale pricing, Long [1999] estimates the volatility of private equity portfolios using outcomes rather than time series.

In this study we apply a method from the real estate finance literature to estimate the characteristics of the economic process from the smoothed process for several alternative asset classes. Whereas in traditional markets the economic process is observable (and reported), in alternative markets stale pricing can prevent observation of the economic process and the smoothed process is reported. Consequently, we use the standard deviations and correlations of the reported smoothed process data to estimate the standard deviations and correlations of the unobservable economic process of returns. The result is a set of risks and correlations that have been adjusted for the effects of stale pricing. For alternative asset classes, we refer to the risk and correlation of the reported smoothed process as the "reported" risk and correlation. We refer to the estimated risk and correlation of the unobservable economic process as the "adjusted" risk and correlation.

This methodology has two distinct advantages over existing methods for estimating risk and correlation in the presence of stale pricing. First, it is computationally simple. Adjusted parameters are calculated directly from historical time series of index returns. This technique is appropriate for individual investments and composites alike, and for liquidated or ongoing positions. Second, this methodology provides a consistent framework for estimating both risk and correlation. This includes correlation between traditional and alternative asset classes, as well as correlation between multiple alternative asset classes.

This article is divided into four sections. In the first section, we discuss the methodology for adjusting risk and correlation for stale pricing. We apply the method to historical private equity and hedge fund returns in the second section. In section three, we discuss the implications of using adjusted parameters for optimal portfolio construction and asset class diversification along the efficient frontier. Finally, we review our conclusions.

METHODOLOGY FOR "DE-LAGGING" RETURNS

Academics and practitioners have developed techniques to deal with stale and appraisal-based pricing in real estate index data. In particular, Geltner [1991] has developed a method for estimating actual volatility by "de-lagging" appraisal-based time series of returns.[1] Like real estate, other alternative asset classes are well suited for this de-lagging process.

One characteristic of private equity and hedge fund index returns that indicates the presence of stale pricing is positive autocorrelation. Autocorrelation can imply that economic valuation events that occur in one time period are priced-in over subsequent periods. Certainly this is true of private equity portfolio companies, which most general partners will revalue only after a significant financing event. It is also true of hedge funds that invest in illiquid or non-exchange-traded securities.

For asset classes subject to stale pricing, some proportion of the current period's actual economic return is realized, some proportion of last period's actual economic return is realized, and some proportion of all previous relevant periods' actual economic returns is realized in the current period. Consequently, using Geltner's model, we define the smoothed return process of asset class *i*, which we denote SR_i, as a weighted average of the economic return process, denoted ER_i. The economic process is continually priced and is independent and identically distributed. The weights in the weighted average that constitutes the smoothed process describe the proportion of each previous period's economic return that is realized in the current period.

As an example, consider security *i* with a continually priced economic return stream (ER_i) with a mean, μ_{ER_i}, of 10% and a standard deviation, σ_{ER_i}, of 20%. If the security in question were subject to stale pricing such that only half of this period's economic price change were realized this period and the other half were realized in the next period, there would exist a corresponding smoothed return process (SR_i), such that:

$$SR_{i,t} = .50 \times ER_{i,t} + .50 \times ER_{i,t-1}$$

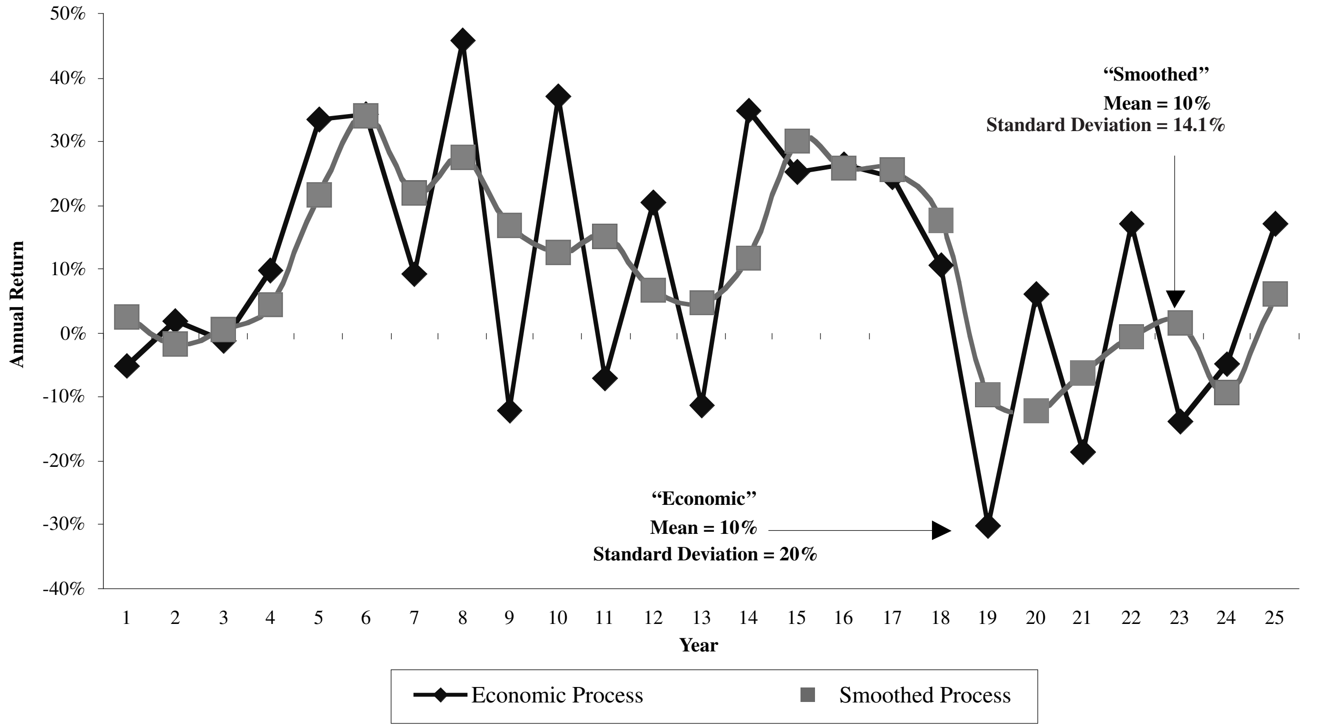
Over a full investment cycle, the mean of the smoothed return stream, μ_{SR_i}, would still equal 10%, but the standard deviation, σ_{SR_i}, would be muted to 14.1%. Exhibit 1 is a graphical representation of this example. For alternative assets, we observe the smoothed return process in reported historical return streams. Our goal is to estimate the characteristics of the underlying economic process, specifically standard deviation and correlation, by adjusting those of the smoothed process.

The general form of the smoothed process example described above is expressed for asset class *i*, with *N* relevant lags, where w_{t-n} refers to weight applied to the *n*th order lag of the economic process, as follows:

$$SR_{i,t} = \sum_{n=0}^{N} w_{i,t-n} \times ER_{i,t-n} \tag{1}$$

EXHIBIT 1

Comparison of Economic Process and Smoothed Process with Two Period Lag



Within this framework, Equation 2 shows that the adjustment to the standard deviation of alternative asset class i, denoted by σ_{ER_i}, can be simplified to multiplication by a scalar. The adjustment factor is a function of the weights in the weighted average smoothed process of asset class i.[2]

$$\sigma_{ER_i} = \sigma_{SR_i} \times \frac{1}{\sqrt{\sum_{n=0}^{N} w_{i,t-n}^2}} \tag{2}$$

We can use this model to estimate the true economic correlation with other asset classes, which is also dampened by stale pricing. As shown in Equation 3, the correlation between the economic process of alternative asset class i and the traditional asset class j, denoted ρ_{ER_i,ER_j}, can be expressed as the correlation between the smoothed process of asset class i and asset class j, denoted ρ_{SR_i,ER_j}, multiplied by a scalar. The adjustment factor for correlation is also a function of the weights in the weighted average of the smoothed process of asset class i.[3]

$$\rho_{ER_i,ER_j} = \rho_{SR_i,ER_j} \times \frac{\sqrt{\sum_{n=0}^{N} w_{i,t-n}^2}}{w_{i,0}} \tag{3}$$

Equation 4 provides a formula for the correlation between two economic processes of alternative asset classes i and j, ρ_{ER_i,ER_j}. Similar to Equations 2 and 3, the adjustment factor for the correlation between alternative asset classes is expressed as the product of the correlation between the smoothed processes of asset classes i and j, ρ_{ER_i,ER_j} and a scalar.[4]

$$\rho_{ER_i,ER_j} = \rho_{SR_i,SR_j} \times \frac{\sqrt{\sum_{n=0}^{N} w_{i,t-n}^2} \times \sqrt{\sum_{n=0}^{N} w_{j,t-n}^2}}{\sum_{n=0}^{N} w_{i,t-n} \times w_{j,t-n}} \tag{4}$$

Equations 2, 3, and 4 constitute a computationally simple method for estimating true underlying risk and correlation of asset classes in the presence of stale pricing from the risk and correlation of the reported smoothed process of returns. Full derivations of the adjustment fac-

EXHIBIT 2
Sources for Historical Asset Class Index Data

Asset Class	Data Source	Data Frequency	Number of Observations (n)	Observation Period
Cash	T-Bill	Monthly	211	January 1985 to August 2002
U.S. Equity	Wilshire 5000	Monthly	211	January 1985 to August 2002
U.S. Bonds	Lehman Aggregate	Monthly	211	January 1985 to August 2002
International Equity	MSCI EAFE	Monthly	211	January 1985 to August 2002
International Bonds	SB WGBI	Monthly	211	January 1985 to August 2002
High Yield Bonds	Lehman CCC	Monthly	211	January 1985 to August 2002
Convertible Arbitrage	CSFB Tremont Index	Monthly	104	January 1994 to August 2002
Dedicated Short Bias	CSFB Tremont Index	Monthly	104	January 1994 to August 2002
Emerging Markets	CSFB Tremont Index	Monthly	104	January 1994 to August 2002
Equity Market Neutral	CSFB Tremont Index	Monthly	104	January 1994 to August 2002
Event Driven	CSFB Tremont Index	Monthly	104	January 1994 to August 2002
Fixed Income Arbitrage	CSFB Tremont Index	Monthly	104	January 1994 to August 2002
Global Macro	CSFB Tremont Index	Monthly	104	January 1994 to August 2002
Equity Long/Short	CSFB Tremont Index	Monthly	104	January 1994 to August 2002
Managed Futures	CSFB Tremont Index	Monthly	104	January 1994 to August 2002
Venture Capital	Cambridge Venture Capital Index	Quarterly	70	January 1985 to June 2002
Buyout & Other	Cambridge Private Equity Index	Quarterly	65	April 1986 to June 2002

tors are found in Geltner, or may be calculated directly by solving for the variance and covariance of Equation 1.

UNSMOOTHING RISK AND CORRELATION IN PRIVATE EQUITY AND HEDGE FUND RETURNS

We applied the de-lagging method described above to historical private equity and hedge fund returns. First we determined the number of relevant periods across which de-lagging is necessary. Next we calculated the weights in the weighted average that makes up the smoothed process. From the weights we then computed the adjustment factors using Equations 2, 3, and 4 and applied them to the smoothed process moments to estimate the moments of the economic process.

The Cambridge Associates LLC U.S. Venture Capital Index tracks quarterly time weighted venture capital returns back to the first quarter of 1981. The Cambridge Associates LLC U.S. Private Equity Index tracks quarterly time weighted buyout, subordinated debt, and special situations returns from the first quarter of 1986. The CSFB/Tremont Hedge Fund Index has monthly historical returns for several hedge fund styles beginning in January of 1994. Some of these indices exhibit the characteristics of stale-pricing induced smoothing: lower than expected absolute risk and correlation with other asset classes, and significant positive autocorrelation. Our sources for historical index data, including traditional asset classes (which are used in the optimizations in section three of this article), are listed in Exhibit 2.

To determine the number of previous periods relevant to the current period's returns, we tested each index for autocorrelation by looking at successive lags. We continued to accept the *n*th lagged period as relevant as long as the *n*th order autocorrelation was statistically significant with 90% confidence. Statistical significance of the autocorrelations was determined using a simple t-test.[5] For hedge funds, which hold mostly public securities, prices and returns are reported monthly and we therefore used monthly returns. For private equity, reporting is done quarterly, so we used quarterly returns.

For venture capital funds, autocorrelations up to the third order were significant, indicating that it takes up to three quarters to pass valuation activity through into prices. Buyouts and other private equity had statistically significant autocorrelations of the first and second order. This difference suggests that, on average, venture capital portfolios are marked-to-market less frequently than buyout portfolios. This result reflects the fact that buyout portfolio companies tend to be more established and "priceable" than venture capital portfolio companies.

EXHIBIT 3

Results of Autocorrelation Significance Tests

	n	Historical Autocorrelation t, t-0	t, t-1	t, t-2	t, t-3	t, t-4	t Statistics t, t-1	t, t-2	t, t-3	t, t-4
Convertible Arbitrage	104	1.00	0.92	0.91	0.89	0.85	6.99***	4.94***	1.69**	1.52*
Dedicated Short Bias	104	1.00	0.05	-0.04	-0.02	-0.10	0.89	-0.78	-0.73	-0.92
Emerging Markets	104	1.00	0.18	-0.02	0.02	0.05	3.21***	0.11	-0.24	-0.72
Equity Market Neutral	104	1.00	0.05	-0.05	0.04	0.08	3.11***	2.01**	0.89	0.14
Event Driven	104	1.00	0.20	0.09	-0.02	-0.08	3.79***	1.42*	0.16	0.14
Fixed Income Arbitrage	104	1.00	0.28	0.08	0.00	0.02	4.45***	1.42*	0.72	1.06
Global Macro	104	1.00	0.57	0.44	0.17	0.15	0.57	0.47	0.86	-1.03
Equity Long/Short	104	1.00	0.09	-0.08	-0.07	-0.09	1.62*	0.55	-0.55	-0.93
Managed Futures	104	1.00	0.30	0.01	-0.02	-0.07	0.43	-0.7	0.09	-0.08
Venture Capital	70	1.00	0.29	0.19	0.09	0.01	5.75***	3.98***	2.56***	-0.36
Buyout & Other	65	1.00	0.35	0.14	0.02	0.01	2.68***	1.34*	-0.55	0.31

* Denotes statistical significance at a 90% confidence level. ** Denotes statistical significance at a 95% confidence level. *** Denotes statistical significance at a 99% confidence level.

Convertible arbitrage hedge funds had significant autocorrelations up to the fourth lag, suggesting that prices are lagged up to four months. Equity market neutral, event-driven, and fixed income arbitrage all had statistically significant autocorrelations of the first and second order. Emerging markets and equity long/short hedge fund styles had significant first order autocorrelations only. Dedicated short bias, global macro, and managed futures did not have statistically significant first order autocorrelations, suggesting that these three hedge fund styles do not have a smoothed process associated with them.

In general, those hedge fund styles that are subject to stale pricing because of illiquidity in the underlying securities (convertible arbitrage and fixed income arbitrage) had a greater number of significant lags. Those that invest primarily in liquid or public market securities (equity long/short, dedicated short bias, global macro, and managed futures) had fewer relevant lags. While the significant autocorrelations cannot necessarily be explained entirely by stale pricing, the de-lagging methodology provides a better estimate of risk than using annualized reported standard deviations in the presence of autocorrelation, regardless of its source.

Also, it was the hedge fund styles with lower observed volatility that have the most significant lags and, therefore, had the largest adjustment factors. The three hedge fund styles with the lowest reported historical volatilities (convertible arbitrage, standard deviation of 5.4%; fixed income arbitrage, standard deviation of 4.3%; and equity market neutral, standard deviation of 3.6%) also had the three highest adjustment factors. Exhibit 3 contains the results of the autocorrelation significance tests.

After finding the appropriate autocorrelations for each alternative asset class, the next step is to solve for the weights in the weighted average that is the smoothed process. The weights in the weighted average are used to adjust the standard deviations and correlations. The weights themselves are found from the solution to a system of equations for the autocorrelations. Appendix A details the process for solving for the weights.

Once the weights are estimated, Equations 2, 3, and 4 are then used to calculate the unsmoothing adjustment factors for standard deviation and correlation. Exhibit 4 shows the weights and the adjustment factors for each of the alternative asset classes. The adjustment factors are applied to the reported historical standard deviations and correlations to compute the adjusted risks and correlations for the alternative asset classes.

Exhibit 5 shows the historical reported and adjusted risks and correlations for each alternative asset class. The venture capital standard deviation increased the most dramatically, from 31% to 59%. The convertible arbitrage and emerging markets standard deviations each increased 5%. Note that the adjustment factors for dedicated short bias, global macro, and managed futures are 1.0, confirming that there is no adjustment for these three. Meaningful changes to correlation included a 0.15 increase to the correlation between venture capital and U.S. public equity, implying that private and public equities are ultimately more similar investments than a cursory examination of the data would suggest. Also, the 0.13 increase to the correlation between convertible arbitrage hedge funds and high yield bonds supports the intuition that convertible arbitrage strategies should exhibit exposure to credit spreads.

EXHIBIT 4

Weights and Adjustment Factors for Alternative Asset Classes

	Estimated Weights					Adjustment Factors	
	w0	w1	w2	w3	w4	For Standard Deviation	For Correlation with Traditional Assets
Convertible Arbitrage	0.41	0.22	0.21	0.06	0.10	191%	128%
Dedicated Short Bias	1.00					100%	100%
Emerging Markets	0.75	0.25				127%	105%
Equity Market Neutral	0.67	0.18	0.15			141%	106%
Event Driven	0.66	0.23	0.11			141%	107%
Fixed Income Arbitrage	0.63	0.26	0.11			144%	109%
Global Macro	1.00					100%	100%
Equity Long/Short	0.86	0.14				115%	101%
Managed Futures	1.00					100%	100%
Venture Capital	0.39	0.19	0.20	0.21		190%	134%
Buyout & Other	0.67	0.20	0.13			140%	106%

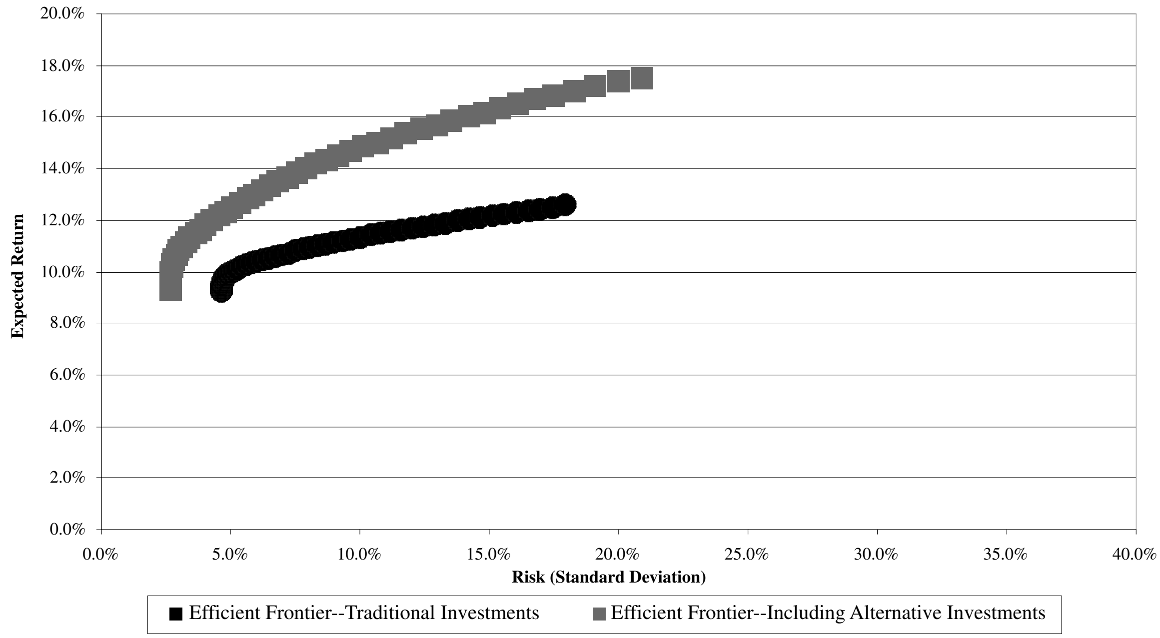
EXHIBIT 5

Raw and Adjusted Risk and Correlation for Alternative Asset Classes

	Convertible Arbitrage	Dedicated Short Bias	Emerging Markets	Equity Market Neutral	Event Driven	Fixed Income Arbitrage	Global Macro	Equity Long/Short	Managed Futures	Venture Capital	Buyout & Other
Reported Historical Risk (Standard Deviation)	5%	19%	20%	4%	7%	4%	15%	13%	13%	31%	9%
Adjusted Risk (Standard Deviation)	10%	19%	26%	5%	10%	6%	15%	15%	13%	59%	13%
Reported Historical Correlations											
with Cash	0.32	0.03	-0.07	0.23	0.15	0.07	0.16	0.13	-0.03	-0.04	-0.09
with U.S. Equity	0.17	-0.84	0.53	0.42	0.63	0.07	0.26	0.72	-0.23	0.43	0.55
with U.S. Bonds	0.11	0.02	-0.07	0.10	0.01	0.14	0.30	0.11	0.22	-0.20	-0.11
with International Equity	0.09	-0.63	0.51	0.33	0.56	0.03	0.12	0.59	-0.08	0.27	0.33
with International Bonds	-0.24	0.07	-0.26	0.05	-0.21	-0.24	-0.20	0.00	0.31	-0.21	-0.17
with High Yield Bonds	0.44	-0.40	0.31	0.28	0.60	0.28	0.10	0.35	-0.38	0.12	0.22
with Convertible Arbitrage	1.00										
with Dedicated Short Bias	-0.24	1.00									
with Emerging Markets	0.33	-0.57	1.00								
with Equity Market Neutral	0.34	-0.39	0.23	1.00							
with Event Driven	0.60	-0.63	0.68	0.40	1.00						
with Fixed Income Arbitrage	0.58	-0.08	0.32	0.07	0.40	1.00					
with Global Macro	0.29	-0.13	0.41	0.22	0.37	0.46	1.00				
with Equity Long/Short	0.27	-0.74	0.58	0.35	0.66	0.20	0.43	1.00			
with Managed Futures	-0.30	0.28	-0.15	0.16	-0.28	-0.17	0.25	-0.09	1.00		
with Venture Capital	0.27	-0.48	0.45	0.29	0.39	0.14	0.16	0.80	-0.37	1.00	
with Buyout & Other	0.32	-0.50	0.30	0.27	0.52	0.20	0.27	0.66	-0.35	0.62	1.00
Adjusted Correlations											
with Cash	0.41	0.03	-0.07	0.25	0.16	0.07	0.16	0.13	-0.03	-0.06	-0.10
with U.S. Equity	0.22	-0.84	0.56	0.44	0.67	0.08	0.26	0.73	-0.23	0.57	0.58
with U.S. Bonds	0.14	0.02	-0.08	0.11	0.01	0.16	0.30	0.11	0.22	-0.27	-0.12
with International Equity	0.12	-0.63	0.53	0.35	0.60	0.04	0.12	0.59	-0.08	0.36	0.35
with International Bonds	-0.30	0.07	-0.28	0.05	-0.23	-0.27	-0.20	0.00	0.31	-0.28	-0.18
with High Yield Bonds	0.57	-0.40	0.32	0.30	0.64	0.31	0.10	0.35	-0.38	0.17	0.24
with Convertible Arbitrage	1.00										
with Dedicated Short Bias	-0.31	1.00									
with Emerging Markets	0.38	-0.60	1.00								
with Equity Market Neutral	0.37	-0.42	0.24	1.00							
with Event Driven	0.65	-0.67	0.69	0.40	1.00						
with Fixed Income Arbitrage	0.62	-0.09	0.32	0.07	0.40	1.00					
with Global Macro	0.37	-0.13	0.43	0.23	0.40	0.51	1.00				
with Equity Long/Short	0.32	-0.75	0.59	0.36	0.67	0.21	0.43	1.00			
with Managed Futures	-0.39	0.28	-0.16	0.17	-0.30	-0.19	0.25	-0.09	1.00		
with Venture Capital	0.29	-0.64	0.55	0.33	0.45	0.16	0.22	1.00	-0.49	1.00	
with Buyout & Other	0.34	-0.53	0.30	0.27	0.52	0.20	0.29	0.67	-0.38	0.71	1.00

EXHIBIT 6

Efficient Frontier Based on Reported Risks and Correlations



IMPLICATIONS FOR EFFICIENT PORTFOLIOS

As shown above, for alternative asset classes subject to stale pricing, reported historical data will understate volatility and the absolute value of correlation with other asset classes. Intuitively one would expect smoothed returns to reflect dampened volatility. Appendix B shows mathematically that, given positive autocorrelations, both the adjusted standard deviation and correlations will be greater in absolute value than or equal to the reported versions. This shows that reported risks and correlations might overstate the diversification benefits of alternative assets.

To test the asset allocation impact of adjusting the moments of alternative asset returns for the effects of stale pricing, we compared two efficient frontiers. The first was optimized based on reported historical risks and correlations and the second was optimized based on adjusted historical risks and correlations. In both cases we considered a hypothetical investor who has a "budget" for alternative assets of no more than 50% of total assets. We applied constraints to reflect the alternative assets budget as well as to prevent leverage and short selling at the asset-class level (no allocations to cash and allocations limited to no less than 0% and no greater than 100% for each asset class). Portfolios on the efficient frontiers were derived using a mean/variance optimization algorithm.

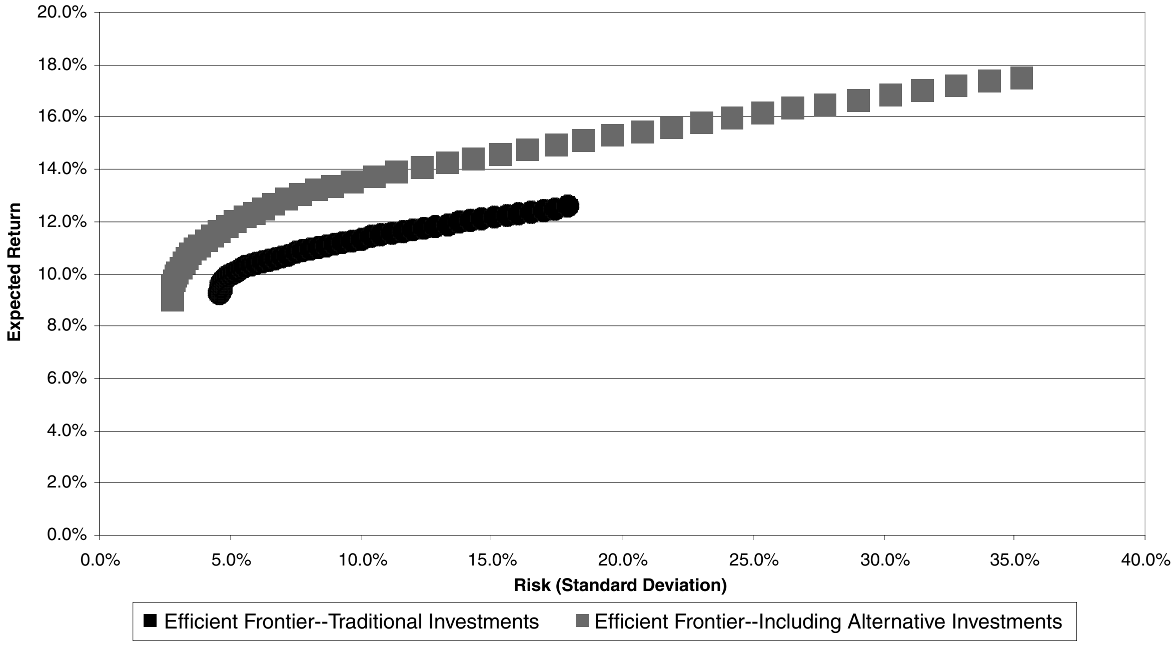
Our goal is to compare the ex-post efficient portfolios over the historical period, so the mean, variance, and correlation inputs to all optimizations were computed directly from the historical returns of the indices summarized in Exhibit 2. We describe the efficient frontier optimized using reported historical risks and correlations as the "reported-risk frontier" and the efficient frontier optimized based on historical risks and correlations adjusted for stale pricing as the "adjusted-risk frontier." Exhibit 6 is a graph of the reported-risk frontier and Exhibit 7 is a graph of the adjusted-risk frontier. For reference, each exhibit also shows an efficient frontier optimized for traditional asset classes only.

The relative positions of the two frontiers confirm that using reported historical moments understates portfolio risk. Compared to the reported frontier, each point along the adjusted frontier has higher risk for each given level of return. That is, the adjusted frontier appears shifted to the right of the reported frontier. On both frontiers the hypothetical investor consumes the entire budget (50% total allocation) for alternative assets at all levels of risk. Although the higher adjusted risk of alternative asset classes is built into the adjusted frontier, the overall appetite for alternative asset classes is not reduced relative to traditional asset classes. Rather, the composition of the traditional portfolio changes to compensate for the higher adjusted risk in alternative investments.

To see how the composition of portfolios changes

EXHIBIT 7

Efficient Frontier Based on Risks and Correlations Adjusted for Stale Pricing



when the effects of stale pricing are removed, we compared three portfolios from each frontier. The three portfolios are: the minimum variance portfolio, the highest return portfolio, and the maximum Sharpe ratio portfolio.[6] Exhibit 8 summarizes the asset allocations of the three portfolios from each frontier. The low-risk portfolio from the reported-risk frontier has a higher return than the low-risk portfolio from the adjusted-risk frontier by 30 basis points, with 10 basis points less risk. The optimizer is "fooled" by stale pricing into allocating to more aggressive asset classes that appear less risky than they actually are. The high-return portfolios from each frontier are identically allocated and have the same return, 17.5%. However, the volatility of this allocation is revealed to be much higher than reported when corrected for the dampening effects of stale pricing. The maximum Sharpe-ratio portfolios are both allocated entirely to bonds in the traditional portfolio. However, the adjusted-risk portfolio is allocated more heavily to hedge funds in the alternative portfolio (80% versus 71.5% on the reported-risk frontier) in order to mitigate risk. The adjusted-risk portfolio has a lower return with slightly more risk.

Incremental allocations to alternative assets are often made with two goals in mind: potential return enhancement and diversification with traditional asset classes. Historically alternatives have certainly provided the former; venture capital, global macro, buyout and other private equity, and equity long/short were the top four performing asset classes in our data set. The latter is less straightforward to assess. One method to measure how well diversified a portfolio is relative to other portfolios is using a statistic called Diversification Benefit (*DB*). The *DB* statistic measures the amount that asset classes in a portfolio have reduced each other's risk. We define *DB* as the difference between the weighted sum of asset class standard deviations and the actual portfolio standard deviation. For a portfolio containing I asset classes with w_i of the portfolio allocated to asset class i and the covariance between asset class i and asset class j equal to $\sigma_{i,j}$:

$$DB = \sum_{i=1}^{I} w_i \times \sigma_i - \sqrt{\sum_{i=1}^{I} w_i \times \sum_{j=1}^{I} w_i \times \sigma_{i,j}}$$

Intuitively, *DB* is the difference between what portfolio risk would be if all asset classes were perfectly correlated and what portfolio risk is, given actual correlations.

For the 50 portfolios on the traditional-only efficient frontier, the average *DB* is 2.0%, indicating that intra-asset correlation has reduced portfolio volatility by that amount each year. If indeed alternative assets are superior diversifiers, we expect a higher average *DB* on the

EXHIBIT 8

Comparison of Three Portfolios from Each Frontier

	LOW RISK PORTFOLIO		MAX SHARPE RATIO PORTFOLIO		HIGH RETURN PORTFOLIO	
	Portfolio from Reported-Risk Frontier	Portfolio from Adjusted-Risk Frontier	Portfolio from Reported-Risk Frontier	Portfolio from Adjusted-Risk Frontier	Portfolio from Reported-Risk Frontier	Portfolio from Adjusted-Risk Frontier
Portfolio Return	9.3%	9.0%	10.8%	10.2%	17.5%	17.5%
Portfolio Risk (Standard Deviation)	2.7%	2.8%	3.0%	3.1%	20.9%	35.3%
Asset Allocation						
Cash	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
U.S. Equity	2.9%	2.7%	0.0%	0.0%	50.0%	50.0%
U.S. Bonds	32.2%	31.8%	41.0%	38.7%	0.0%	0.0%
International Equity	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
International Bonds	11.2%	12.8%	9.0%	11.3%	0.0%	0.0%
High Yield Bonds	3.8%	2.7%	0.0%	0.0%	0.0%	0.0%
Convertible Arbitrage	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Dedicated Short Bias	9.2%	13.8%	3.0%	6.5%	0.0%	0.0%
Emerging Markets	2.2%	0.7%	0.0%	0.0%	0.0%	0.0%
Equity Market Neutral	20.3%	15.3%	32.7%	29.5%	0.0%	0.0%
Event Driven	0.0%	7.5%	0.0%	4.0%	0.0%	0.0%
Fixed Income Arbitrage	11.2%	7.4%	0.0%	0.0%	0.0%	0.0%
Global Macro	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Equity Long/Short	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Managed Futures	0.0%	1.8%	0.0%	0.0%	0.0%	0.0%
Venture Capital	1.0%	2.3%	2.3%	1.0%	50.0%	50.0%
Buyout & Other	6.0%	1.4%	12.0%	9.0%	0.0%	0.0%

frontiers that include allocations to alternatives. This is the case; the average *DB* along the reported-risk frontier is 10.9%, suggesting a significant increase in the diversification within the portfolio with the introduction of alternative asset classes. However, the average *DB* along the adjusted-risk frontier is 5.1%, suggesting that over half of the benefits from diversification associated with alternative assets may in fact be illusory—derived from the effects of stale pricing.[7]

CONCLUSION

Reported historical data series do not necessarily reflect the true risk and correlation of alternative assets. In the cases of private equity and hedge funds it is often necessary to adjust the reported risks and correlations for the effects of stale pricing. Once this is accomplished, investors are better able to apply traditional mean/variance optimization tools to aid in the asset allocation process. Using historical data as an example, we have found that adjusting risk and correlation for stale pricing will substantially increase the perceived risk of alternative asset classes and decrease the diversification benefits of alternative assets with most traditional asset classes, in our example eliminating half of the diversification benefit associated with allocating to alternative assets. Despite this, we find that optimal portfolios based on adjusted risk and correlation do not have a lower overall allocation to alternative assets. Although the overall allocation decision still includes alternatives, the composition of the traditional and alterative portfolios is adjusted to better manage risk. In other words, adjusting for the effects of stale pricing has a meaningful effect on the perception of risk and the asset allocation decision.

APPENDIX A

Process for Solving for Weights

For $N + 1$ relevant autocorrelations (including the contemporaneous return/or zero lag autocorrelation), there is a system of N equations that define each autocorrelation of order greater than zero order. For asset class i, subject to stale pricing, the smoothed return at time t, denoted $SR_{i,t}$, is a function of N previous economic returns, denoted $ER_{i,t-n}$ for the nth previous period, and the weights associated with each economic return ($w_{i,t-n}$):

$$SR_{i,t} = \sum_{n=0}^{N} w_{i,t-n} \times ER_{i,t-n}$$

The variance of the smoothed return ($\sigma^2_{SR_i}$) is a function of the variance of the economic return ($\sigma^2_{ER_i}$):

$$\sigma^2_{SR_i} = \sum_{n=0}^{N} w^2_{i,t-n} \times \sigma^2_{ER_i}$$

and the kth order serial covariance ($\sigma_{SR_{i,t},SR_{i,t-k}}$) of the smoothed return is:

$$\sigma_{SR_{i,t},SR_{i,t-k}} = \sigma^2_{ER_i} \times \sum_{n=0}^{N-k} w_{i,t-n} \times w_{i,t-n-k}$$

So, the system of N equations for each autocorrelation described by:

$$\rho_{SR_{i,t},SR_{i,t-k}} = \frac{\sum_{n=0}^{N-k} w_{i,t-n} \times w_{i,t-n-k}}{\sum_{n=0}^{N} w^2_{i,t-n}}$$

for $k = 1$ to N.

And the final equation to make the system well specified is the wealth constraint on the weights that ensures that the mean of the economic process will equal the mean of the smoothed process.

$$\sum_{n=0}^{N} w_{i,t-n} = 1$$

The system of equations is solved simultaneously to find the vector of weights ($w_{i,t-n}$), which can be used in Equations 1, 2, and 3 to find the adjustment factors for standard deviation and correlation.

APPENDIX B

Demonstration That the Standard Deviation and Absolute Value of Correlations of the Economic Process Are Greater Than Those of the Smoothed Process

For asset class i, subject to stale pricing, from Equation 1:

$$\frac{\sigma_{ER_i}}{\sigma_{SR_i}} = \frac{1}{\sqrt{\sum_{n=0}^{N} w^2_{i,t-n}}}$$

Since the weights must sum to unity,

$$0 \le w_{i,t-n} \in n$$

and it follows that

$$w^2_{i,t-n} \le w_{i,t-n} \in n$$

and therefore

$$0 \le \sum_{n=0}^{N} w^2_{i,t-n} \le 1$$

and

$$0 \le \sqrt{\sum_{n=0}^{N} w^2_{i,t-n}} \le 1$$

so

$$\frac{1}{\sqrt{\sum_{n=0}^{N} w^2_{i,t-n}}} \ge 1$$

For correlation, from Equation 2:

$$\frac{\rho_{ER_i}}{\rho_{SR_i}} = \frac{\sqrt{\sum_{n=0}^{N} w^2_{i,t-n}}}{w_{i,0}}$$

$$= \sqrt{1 + \frac{\sum_{n=1}^{N} w^2_{i,t-n}}{w^2_{i,0}}}$$

As shown above,

$$0 \le \sum_{n=1}^{N} w^2_{i,t-n} \le 1$$

and

$$0 \le w_{i,0}^2 \le 1$$

so it follows that

$$\frac{\sum_{n=1}^{N} w_{i,t-n}^2}{w_{i,0}^2} \ge 0$$

and therefore

$$\sqrt{1+\frac{\sum_{n=1}^{N} w_{i,t-n}^2}{w_{i,0}^2}} \ge 1$$

ENDNOTES

[1]For this study we are using Geltner's "noiseless" model, with univariate weight estimation, assuming that the number of constituents in our private equity and hedge fund indices is large enough to make random appraisal error insignificant.

[2]This equation is a version of Geltner's Equation 10.

[3]This equation is derived from a special case of Geltner's Equation 12, where asset *j* is not a real estate asset, but continually priced without lag.

[4]This equation is derived directly from Geltner's Equation 12.

[5]Where *r* is the sample correlation coefficient and *n* is the number of observations in the sample, the t-statistic for a correlation coefficient is computed as follows:

$$t = \frac{r \times \sqrt{n-2}}{\sqrt{1-r^2}}$$

[6]The Sharpe ratio is defined as the ratio of excess return over the risk-free rate to standard deviation. For this study we used the historical average one-year constant maturity Treasury bond return as a proxy for the risk-free rate. Where R_p is the expected return of the portfolio and R_{rf} is the risk-free rate of return and σ_p is the standard deviation of portfolio returns, Sharpe ratio is expressed as follows:

$$SharpeRatio = \frac{R_P - R_{rf}}{\sigma_P}$$

[7]The results of this study, including the absolute level of risk associated with each asset class, the degree of correlation between asset classes, and the suggested reduction in diversification benefits associated with alternative assets when adjusting for the effects of stale pricing, are all based upon the specific historical time period examined in the analysis. Results may differ when other time periods are used.

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